Ch. 3: Simple Resistive Circuits

Analytical Box so far contains:
- Ohm's Law
- Kirchoff's current and voltage laws.
- Tools of this chapter will allow us to tackle more complex interconnections.

- Terminology: dc current source means dc constant current source, and dc voltage source means dc constant voltage source.

Practical Perspective: A rear window defroster.

3.1 Resistors in Series

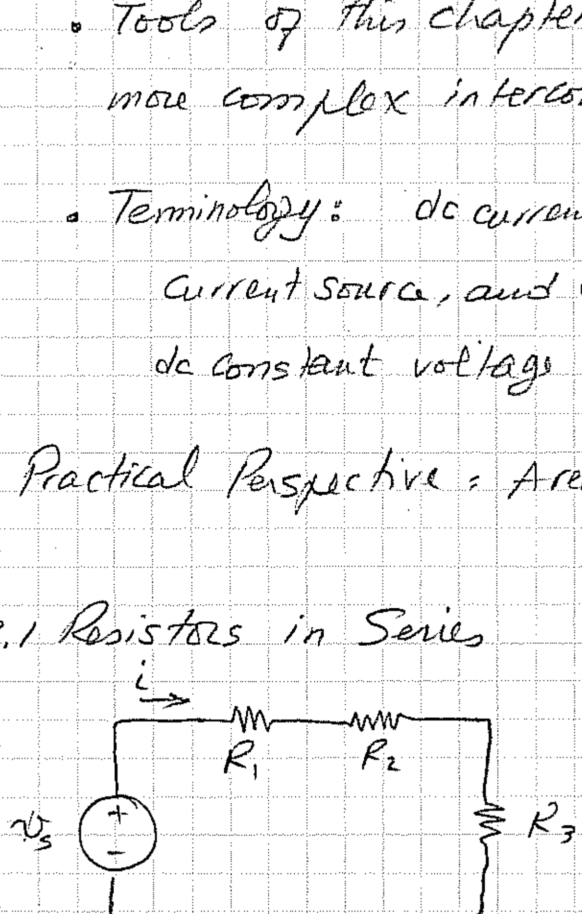


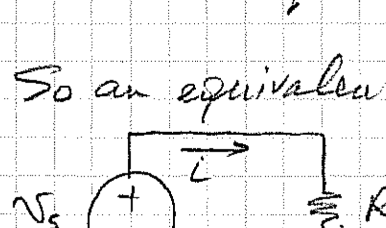
Fig. 1.

KVL: $v_s = iR_1 + iR_2 + iR_3 \Rightarrow$

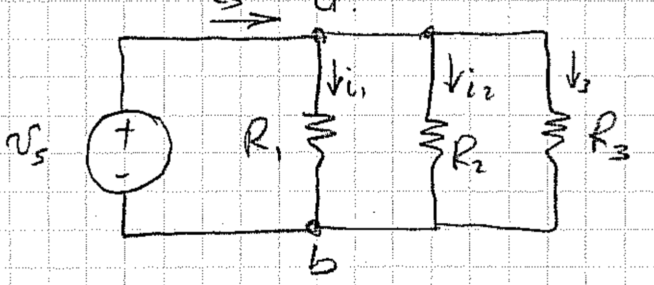
$$v_s = i(R_1 + R_2 + R_3) = i R_{eq}$$

where $R_{eq} = R_1 + R_2 + R_3$

So an equivalent Circuit would be:



3.2 Resistors in Parallel



KCL at node a:

$$i_s = i_1 + i_2 + i_3 \qquad (1)$$

Resistances share the same voltage:

$$v_s = R_1 i_1 = R_2 i_2 = R_3 i_3$$

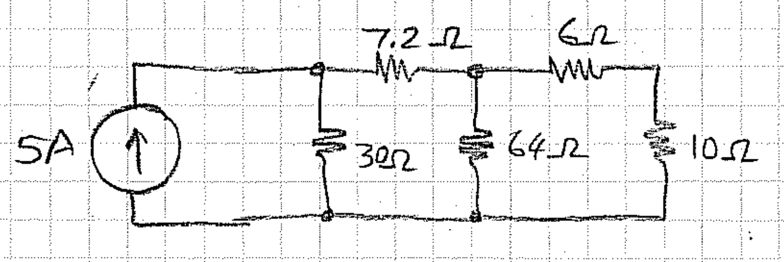
so $\quad i_1 = \dfrac{v_s}{R_1}, \quad i_2 = \dfrac{v_s}{R_2}, \quad i_3 = \dfrac{v_s}{R_3}$

replace in KCL above:

$$i_s = v_s \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) \Rightarrow$$

$$i_s = v_s \frac{1}{R_{eq}} \Rightarrow \quad R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}$$

Assessment Problem 3.1



a) find v

b) Power delivered by the 5A source

c) Power dissipated in 10 Ω resistor.

Solution:

a) i) Combine the 6Ω and 10Ω in series:

$$R_1 = 6+10 = 16\Omega$$

ii) Combine $R_1 = 16\Omega$ in parallel with the 64Ω

$$R_2 = \frac{16 \times 64}{80} = 12.8\Omega$$

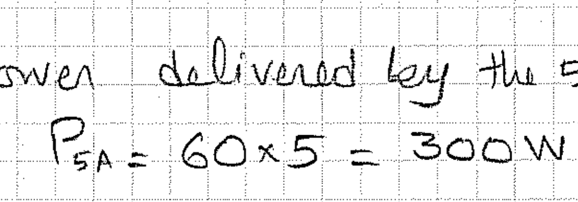
iii) Now $R_2 = 12.8\Omega$ is in series with the 7.2Ω resistor:

$$R_3 = 12.8 + 7.2 = 20\Omega$$

iv) Finally $R_3 = 20\Omega$ is in parallel with the 30Ω resistor:

$$R_4 = \frac{20 \times 30}{20+30} = 12\Omega$$

So the final equivalent circuit is:



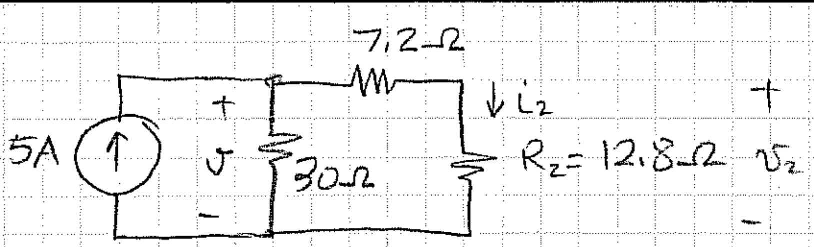
So $v = 5 \times 12 = 60V$.

b) Power delivered by the 5A source:

$$P_{5A} = 60 \times 5 = 300W.$$

c) Power dissipated in the 10Ω resistor:

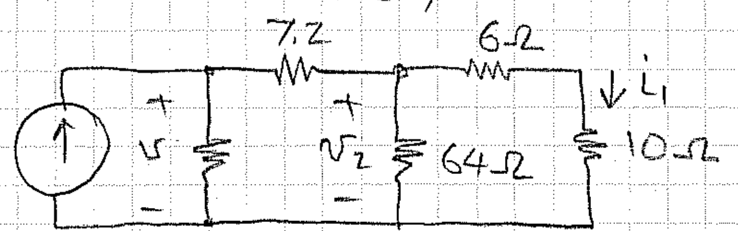
i) Find the current in R_2 which is the same at that of the current in the 7.2Ω:



$$i_2 = \frac{v}{7.2 + 12.8} = \frac{60}{20} = 3\,A$$

The voltage $v_2 = R_2\, i_2 = 12.8 \times 3 = 38.4\,V$

Now the current in R_1 or in the 10Ω resistance can be found:

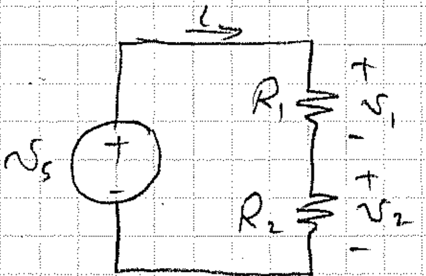


$$i_1 = 38.4 / (16) = 2.4\,A.$$

Power dissipated in 10Ω resistance:

$$P_{10\Omega} = (2.4)^2 \times 10 = 57.6\,W$$

3.3 The Voltage-Divider and Current-Divider Circuits.



KVL in loop:

$$v_s = v_1 + v_2$$
$$= iR_1 + iR_2$$

So $i = \dfrac{v_s}{R_1 + R_2}$

Using Ohm's law on R_1 and R_2:

$$v_1 = iR_1 = v_s \frac{R_1}{R_1 + R_2}$$

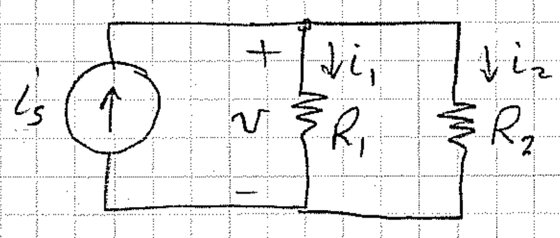
v_{omax} occurs when R_2 is 10% high and R_1 is 10% low:

$$v_{omax} = 100 \frac{110}{110+22.5} = 83.02\,V$$

v_{omin} occurs when R_1 is 10% low and R_2 is 10% high:

$$v_{omin} = 100 \frac{90}{90+27.5} = 76.60\,V$$

The Current Divider Circuit:



The resistances share the same voltage:

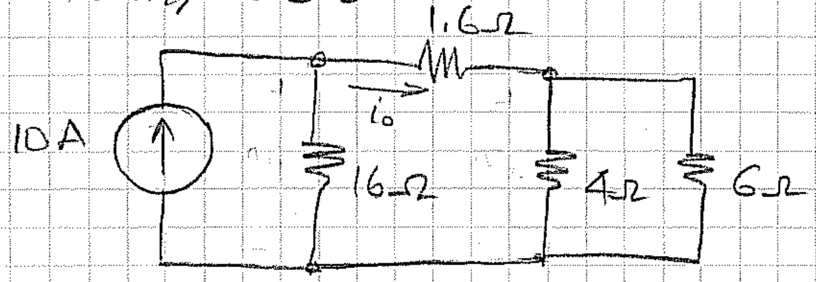
$$v = R_1 i_1 = R_2 i_2 = R_{eq} i_s = \frac{R_1 R_2}{R_1 + R_2} i_s$$

From equation above:

$$R_1 i_1 = \frac{R_1 R_2}{R_1 + R_2} i_s \Rightarrow i_1 = \frac{R_2}{R_1 + R_2} i_s$$

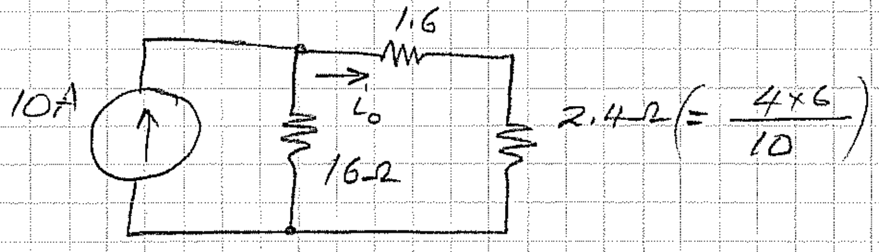
$$R_2 i_2 = \frac{R_1 R_2}{R_1 + R_2} i_s \Rightarrow i_2 = \frac{R_1}{R_1 + R_2} i_s$$

Example 3.3



Find power dissipated in the 6Ω resistor.

The above circuit simplifies to:



$$i_o = 10 \frac{16}{16+4} = 8A$$

Now the current i_o divides between the 4Ω and 6Ω resistors. The current in the 6Ω resistor is:

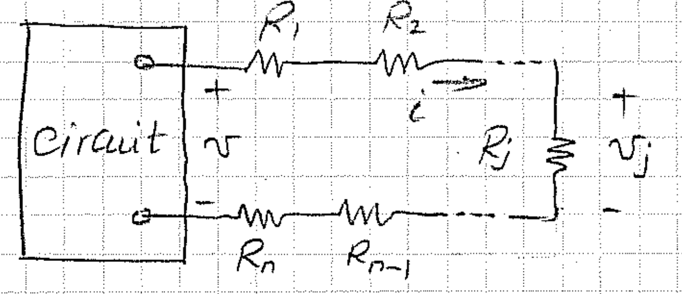
$$i_{6\Omega} = 8 \times \frac{4}{10} = 3.2A.$$

$$P_{6\Omega} = 6 \times (3.2)^2 = 61.4 \text{ W.}$$

3.4 Voltage and Current Division

This is a generalization of the rules developed in the previous section:

Voltage Division:



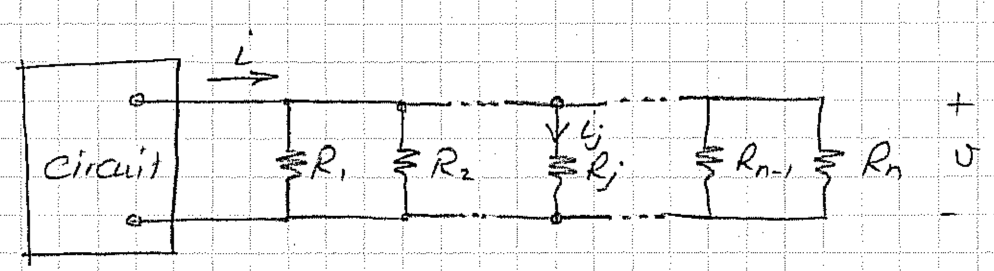
All resistances have the same current

$$i = \frac{v}{R_{eq}} \qquad R_{eq} = \sum_{j=1}^{n} R_j$$

The voltage v_j is given by:

$$\boxed{v_j = i R_j = v \frac{R_j}{R_{eq}}}$$

Current Division:



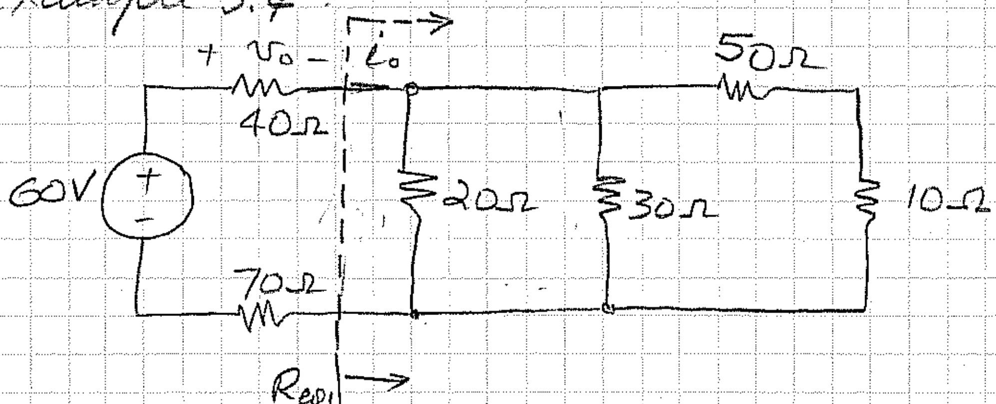
All resistances have the same voltage v:

$$v = i R_{eq} \qquad R_{eq} = \left(\sum_{j=1}^{n} \frac{1}{R_j} \right)^{-1}$$

The current in resistance R_j is given by:

$$i_j = \frac{v}{R_j} = \frac{R_{eq}}{R_j}\, i$$

Example 3.4:



a) Use voltage division to determine v_o

b) Find i_o and use current division to determine current in the 30Ω resistor

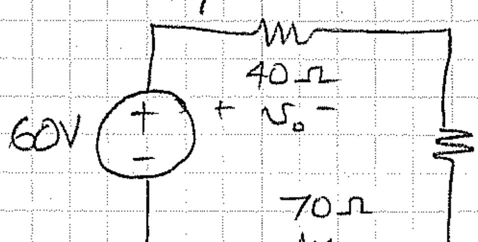
c) Find power dissipated in the 50Ω resistor.

Solution:

a) Find R_{eq_1}:

$$R_{eq_1} = 20 \,//\, 30 \,//\, (50+10)$$
$$= \left(\frac{1}{20} + \frac{1}{30} + \frac{1}{60}\right)^{-1} = 10\,\Omega$$

The equivalent circuit is now:



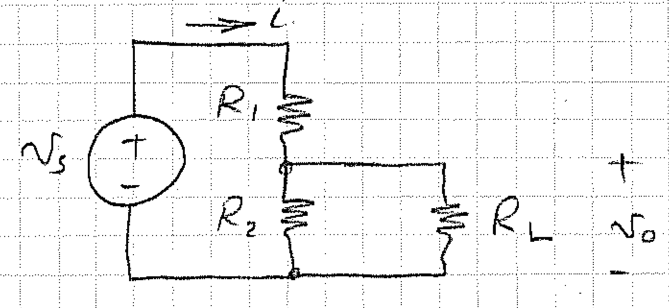
50

$$v_o = 60 \times \frac{40}{(40+70+10)} \Rightarrow$$

$$v_o = 20V.$$

$$v_2 = i R_2 = v_s \frac{R_2}{R_1 + R_2}$$

A voltage divider connected to a load resistor R_L:



In the circuit above find v_o:

i) Find the equivalent resistance of $R_2 // R_L$:

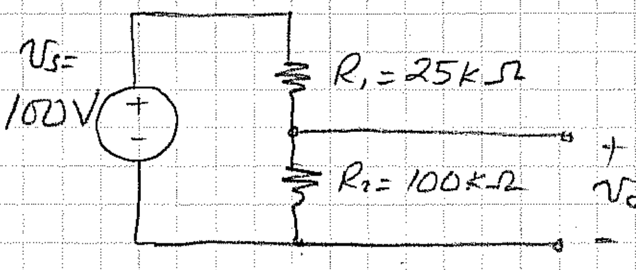
$$R_{eq} = R_2 // R_L = \frac{R_2 R_L}{R_2 + R_L}$$

ii) The voltage v_o is found using the current divider rule:

$$v_o = v_s \frac{R_{eq}}{R_1 + R_{eq}}$$

As $R_L \rightarrow \infty \Rightarrow R_{eq} \longrightarrow R_2$

Example 3.2:



The tolerance on the values of R_1 and R_2 is $\pm 10\%$.

Find the maximum and minimum value of v_o

b)
$$i_o = \frac{v_o}{40} = \frac{20}{40} = 0.5A.$$

The current in the 30 Ω resistor is given by:

$$i_{30} = \frac{R_{eq_1}}{30} i_o = \frac{10}{30} \times 0.5 = 0.1667\ A$$
$$= 166.7\ mA$$

c) $P_{50\Omega}$:

The current in the 50Ω resistor is given by:

$$i_{50} = \frac{R_{eq_1}}{(50+10)} i_o = \frac{10}{60} \times 0.5 = 0.08333\ A$$
$$= 83.33\ mA.$$

$$P_{50\Omega} = 50\, i_{50}^2 = 50 \times 0.0833^2$$

$$= 0.3472\ W = 347.2\ mW.$$

3.5 Measuring Voltage and Current:

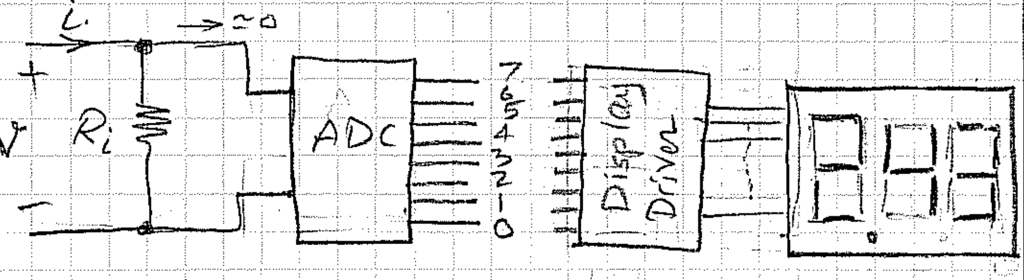
Two methods of measuring:

- digital (used in modern equipment).
- Analog (historical; uses the d'Arsonval movement) used up until the early 1970's.

The digital meter measures the voltage across a resistance and converts the signal to a digital signal using an

FE 1

analog to digital converter (ADC):



Range of v: 10V
Range of i: 0.1 mA

```
7 6 5 4 3 2 1 0

0 0 0 0 0 0 0 0      0V
0 0 0 0 0 0 0 1      ___
0 0 0 0 0 0 1 0
0 0 0 0 0 0 1 1
0 0 0 0 0 1 0 0          2^8 values or
                         2^8-1 intervals = 255
. . . . . . . .
1 1 1 1 1 1 0 0
1 1 1 1 1 1 0 1
1 1 1 1 1 1 1 0
1 1 1 1 1 1 1 1      9.99
```

each interval has voltage increment q:

$$\Delta V = \frac{9.99}{2^8-1} = \frac{9.99}{255} = 0.039218\ V$$

So if the output of ADC is:

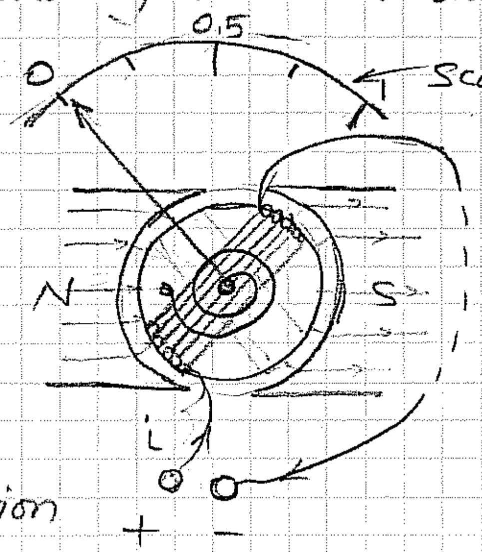
```
7 6 5 4 3 2 1 0
0 0 1 1 1 1 1 0
```

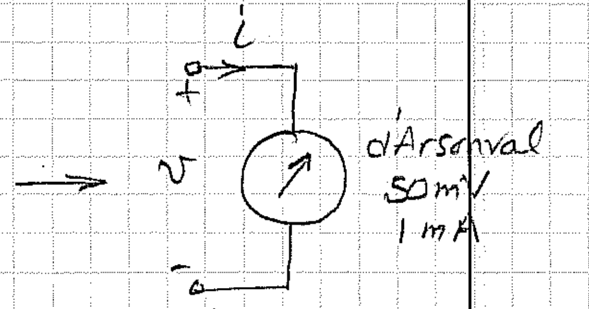
$\rightarrow (2^5+2^4+2^3+2^2+2) \times 0.039218$

$= 62 \times 0.039218 = 2.43$

So the display is 2.43 V ...

The analog meter is based on the d'Arsonval Movement





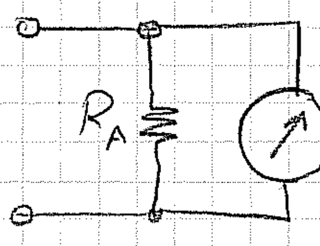
Operation

* Current flows in coil creates a torque proportional to current
* The torque causes the coil to rotate
* The spring deforms and creates a balancing torque

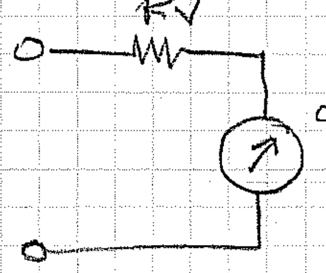
* For a given movement a current of 1 mA causes the pointer to move 1 full scale. If The voltage applied is 50 mA, it means that the internal resistance is 50 Ω

* So when the current entering is 0.5 mA, the pointer is at midscale and the applied voltage is $0.5 \times 50 = 25$ mV.

We can construct an ammeter using the d'Arsonval movement by adding a resistor across it as shown



A voltmeter is constructed by adding a resistance R_V in series with the d'Arsonval movement as shown:



Example 3.5: d'Arsonval Ammeter

a) what should R_A be for a fullscale reading of 10mA.

The current in R_A is $10-1=9\,mA$

Since the voltage is 50mV, then

$$R_A = \frac{50}{9} = 5.55\,\Omega$$

b) What should R_A be for a full scale reading of 1 A

$$R_A = \frac{50}{1000-1} = 50.05\ m\Omega$$

c) How much resistance is added to the circuit when the 10 mA ammeter is added to the circuit.

$$R_m = \frac{50\ mV}{10\ mA} = 5\Omega$$

Example 3.6: d'Arsonval Voltmeter

a) What should R_v for a full-scale reading of 150V?

Full scale requires 50mV across the movement with a current of 1mA.

$$(R_v + 50) \times 1 \times 10^{-3} = 150 \Rightarrow$$

$$R_v = \frac{150}{10^{-3}} - 50 = 149950\Omega$$

b) for a full scale reading of 5V:

$$(R_v + 50) \times 10^{-3} = 5 \Rightarrow$$

$$R_v = \frac{5}{10^{-3}} - 50 = 4950\ \Omega$$